The influence of horizontal vibratory excitation on the behavior of a nanofluid in natural convection

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Abstract – The objective of the present work is to study the effect of horizontal vibratory excitation on the behavior of a water-Al₂O₃ nanofluid in natural convection in a cavity. The nanofluid used is composed of aluminum oxide nanoparticles in suspension of water, is provided at various volume fractions. The two horizontal walls are isothermal, maintained at cold temperature $T_c$ and heat temperature $T_h$. The two vertical walls are adiabatic. The basic equations describing the flow driven by natural convection consist of mass conservation, momentum and energy. For the physical parameters of Al₂O₃-water nanofluid, we use the Brinkman and Wasp model. Transport equations are solved numerically by finite element method. Results are presented in the form of streamlines, isotherms and the flux ratio.

Keywords: Nanofluid, natural convection, excitation vibratory, Rayleigh number

I. Introduction

Nanofluids are liquid suspensions of nano-sized particles. These new materials have attracted much attention since anomalously large enhancements in effective thermal conductivities were reported over a decade ago [1–4]. In the literature, a rich and variety of numerical results have been published on the phenomenon of natural convection in differentially heated shallow enclosures with various wall conditions were carried out by Acharya et al. [5]–[6], Cozia et al. [7], Facas [8], Lakhal et al. [9], Shi et al. [11] and Keblinsky et al. [10]. Vibrations are known to be among the most effective ways of affecting the behavior of fluid systems in the sense of increasing or reducing the convective heat transfer. They do not exist many studies in the field of vibration for nanofluids [14-18], [19-23], [24-27].

In this study we will analyze the natural convection heat transfer in square enclosures under horizontal vibratory excitation. The objective of this work is to study the influence of vibration on natural convection of nanofluids.

II. Physical Model and mathematical formulation

II.2.1. Physical Model

Figure 1 shows the physical model of the problem and its boundary conditions. It consists of a two dimensional square enclosure of height and width $H$. In the present analysis, Cartesian coordinate system will be applied to the enclosure. The temperatures of the vertical walls have been considered to be insulated. The top and the bottom horizontal walls are maintained at cold temperature $T_c$ and heat temperature $T_h$ at $y=0$ and $y=H$ respectively. The nanofluid is Newtonian, incompressible, and the flow is laminar. The nanofluid (Al₂O₃-water) is submitted under horizontal vibratory excitation. Moreover, it is assumed that both fluid phase and nanoparticles are in thermal equilibrium state and they flow at the same velocity. The Boussinesq approximation is assumed to be valid.
II.2. Mathematical formulation

The governing equations for a steady, two-dimensional flow are as follows:

- **Continuity:**
  \[
  \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
  \]

- **X-Momentum:**
  \[
  \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf,0}} \frac{\partial p}{\partial x} + \frac{\mu_{eff}}{\rho_{nf,0}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - (1 - \beta_{nf} \Delta T)bw^2 \sin(\omega t) \tag{2}
  \]

- **Y-Momentum:**
  \[
  \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf,0}} \frac{\partial p}{\partial y} + \frac{\mu_{eff}}{\rho_{nf,0}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - (1 - \beta_{nf} \Delta T)g \tag{3}
  \]

- **Energy:**
  \[
  \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
  \]

- **Stream function equation:**
  \[
  \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \tag{5}
  \]

Where

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho p)_{nf}} \tag{6}
\]

The effective thermal conductivity of nanofluids has been determined by the model proposed by Wasp [12].

\[
K_{nf} = K_f \frac{(2-2\varphi)k_f + (1+\varphi)K_s}{(2+\varphi)k_f + (1-\varphi)K_s} \tag{7}
\]

The effective density of a fluid containing suspended particles at a reference temperature is given by:

\[
\rho_{nf,0} = (1 - \varphi)\rho_{f,0} + \varphi \rho_{s,0} \tag{8}
\]

The effective viscosity of a fluid containing a dilute suspension of small rigid spherical particles is given by Brinkman [13] as:

\[
\mu_{eff} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{9}
\]

The heat capacitance of the nanofluid can be calculated as:

\[
(TC)_{nf} = (1 - \varphi)(TC)_{f} + \varphi(TC)_{s} \tag{10}
\]

The ratio of the flux is given by:

\[
\frac{q_n}{q_f} = \frac{2 \pi (1 + \varphi)(\rho p_{nf})}{R_k + (\frac{2\pi}{3})(1-\varphi)} \frac{\partial T}{\partial y} \tag{11}
\]

Avec

\[
R_k = \frac{k_s}{k_f}
\]

Based upon the previous assumptions and introducing the following dimensionless variables,

\[
(x', y') = (\frac{x}{H}, \frac{y}{H}), (u', v') = (\frac{u}{H}, \frac{v}{H}), T' = \frac{(T - T_c)}{(T_h - T_c)}, P' = \frac{PH^2}{\rho_{nf,0} \alpha_f}, T^* = \frac{H^2}{\alpha_f}, \omega = \frac{H}{\alpha_f}
\]

The governing equations for the problem in dimensionless form are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{12}
\]

\[
\frac{1}{1 - \varphi + \varphi R_p} \left( \frac{\partial u}{\partial \tau} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) = -\frac{\partial P^*}{\partial x} + \frac{\partial \Pr}{\partial y} \frac{\partial u}{\partial y} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \tag{13}
\]

\[
\frac{1}{1 - \varphi + \varphi R_p} \left( \frac{\partial v}{\partial \tau} + \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \right) = -\frac{\partial P^*}{\partial y} + \frac{\partial \Pr}{\partial x} \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \frac{\partial v}{\partial x} \tag{14}
\]
\[
\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \left[ 2 + \left( 1 + \frac{\phi}{1 - \phi} \right) R_y \right] \left[ \frac{1}{R_s} + \left( \frac{2 + \phi}{2 - \phi} \right) \right] \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \tag{15} \]

Where
\[ R_k = \frac{k_s}{k_f} \]

The expressions for dimensionless parameters are given as:
\[ \frac{P_T}{\rho_f c_f} \text{Ra} = \frac{\rho_f g \beta_f H^2 \Delta T}{\mu_f \sigma_f}, R_\rho = \frac{\rho_s}{\rho_f}, R_\beta = \frac{\beta_s}{\beta_f}, R = \frac{Ra}{Ra} \]

Dimensionless boundary conditions are illustrated in table 1.

<table>
<thead>
<tr>
<th>Walls of cavity</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom walls</td>
<td>( u^* = v^* = 0 \text{ and } T^* = 1 )</td>
</tr>
<tr>
<td>Top walls</td>
<td>( u^* = v^* = 0 \text{ and } T^* = 0 )</td>
</tr>
<tr>
<td>Vertical walls</td>
<td>( u^* = v^* = 0 \text{ and } \frac{\partial T^<em>}{\partial x^</em>} = 0 )</td>
</tr>
</tbody>
</table>

Table 1. Dimensionless boundary conditions

III. Results and Discussions

The numerical method used to solve the governing equations (11)-(14) is the finite element method. The computational domain consists of bi-quadratic elements which correspond to \( 41 \times 41 \) grid points and a Lagrange quadratic interpolation has been chosen. The thermo physical properties of nanofluid \( \text{Al}_2\text{O}_3 + \text{water} \) is schown in table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Literature values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr_1</td>
<td>7</td>
</tr>
<tr>
<td>R_ρ</td>
<td>3.9773</td>
</tr>
<tr>
<td>R_β</td>
<td>0.02877</td>
</tr>
<tr>
<td>R_s</td>
<td>61.95</td>
</tr>
<tr>
<td>Ra</td>
<td>( 10^3 )</td>
</tr>
</tbody>
</table>

Table 2. Thermophysical properties of nanofluid \( \text{Al}_2\text{O}_3 + \text{water} \)

The numerical results presented and discussed are obtained in a period \( 0 \leq \text{wt} \leq 2\pi \). The range of the volume fraction \( \phi \) used in this study varied between \( 0 \leq \phi \leq 10\% \). The Rayleigh number is \( \text{Ra} = 10^3 \) and the Rayleigh number ratio \( \text{R} = 50 \). Figure 2 and 3 plots the streamlines and the isotherms in the absence of vibrational excitation (a) and under the influence of vibrational excitation (b) for the values of the volume fraction \( \phi = 0.04, \phi = 0.1 \).

Figure 2. Isotherms and Streamlines for \( \phi = 0.04 \) : (a) absence of vibrational excitation, (b) presence of vibrational excitation

Figure 3. Isotherms and Streamlines for \( \phi = 0.1 \) : (a) absence of vibrational excitation, (b) presence of vibrational excitation

In the absence of vibrational excitation for \( \text{Ra} = 10^3 \), the conduction is dominant giving functions of near-zero. For the same Ra value, and under the influence of vibratory excitation, Figure 2 there are two areas: in the first half period \( (0 \leq \text{wt} \leq \pi) \) shows thermal stratification of the isotherms near the horizontal walls, which promotes conduction. At the heart of the cavity, the isothermal change of pace allows us to say that the dominant mode of transfer is convection. The flow structure is presented as a single cell that circulates in the direction of treble-wise. In the second half period \( (\pi \leq \text{wt} \leq 2\pi) \), the flow is reversed as shown by the evolution of the isotherms and streamlines. By
increasing the volume fraction (Figure 3) in the first half period \(0 \leq \omega t \leq \pi\), the thickness of the boundary layer widens, thereby reducing the intensity of convective transfer. The flow is always represented as a single cell, or the vortex of the streamline at the heart of the cavity narrowed thereby showing that the force of movement of the flow decreases. By comparing cases (a) and (b) of the two figures, one can deduce that the vibrational excitation influences both the mode of transfer and the flow structure. Thus, the variation of the volume fraction implies a change of values of temperature and stream function values.

Figures 2, 4, 5 and 6 illustrate the evolution of the isotherms and streamlines for \(\phi = 0.04\), \(w = 25\) at different values of the ratio \(R\). For Figure 4, in the first half period \(0 \leq \omega t \leq \pi\), the isotherms exhibit thermal stratification near the horizontal walls, whose conduction is dominant. At the heart of the cavity, the isotherms are deformed giving rise to convection. For the current lines, presented as single cell or flow flows in the direction of treble-wise. In the second half period \(\pi \leq \omega t \leq 2 \pi\), the flow is reversed as shown by the isotherms and streamlines. By increasing the value of \(R\), the horizontal boundary layer decreases, which shows that convection is the most dominant transfer mode. For disposal, the curves show a single cell structure. By increasing the value of \(R\), the shape of the cells is deformed around the vortex until two cells appear at the heart cavity when \(R = 150\) and \(R = 200\). Therefore, increasing the ratio \(R\) leads to a change in the mode of transfer and flow structure.

Figures 3, 7, 8, 9 and 10 show the distribution of isotherms and streamlines for \(\phi = 0.04\), \(R = 50\) at different values of the pulse \(w\). By increasing the value of the pulse, the boundary layer expands, showing that the dominant transfer mode is conduction. In addition, the flow structure changes in shape.
Figure 11 illustrates the variation of the ratio of the flux depending on the volume fraction, the pulsation, and the Rayleigh number ratio. In Figure 11-a, the ratio $Q_{nf}/Q_f$ increases with increasing volume fraction. Also, the ratio of the flux increases with the increase of the pulse (Fig. 11-b) and the ratio $R$ (Fig. 11-c).

**Figure 11.** Variation of the ratio of the flux depending on:

(a) volume fraction, (b) pulsation, (c) Rayleigh number ratio.

### IV. Conclusion

The aim of present work is to study the effect of vibration on the flow of a nanofluid $\text{Al}_2\text{O}_3$+water in a closed cavity. The results obtained indicate that:

1. Horizontal vibrational excitation gives rise to convection although $Ra=10^3$ corresponds to conduction in the absence of vibration.

2. Horizontal vibrational excitation influences the flow structure, the values of stream functions and temperature.

3. The ratio of the flux increases with the increase of the volume fraction, the pulse and the Rayleigh number ratio.

### References


[16] Fu, W.S., Shieh, W.J. A study of thermal-convection in an enclosure induced simultaneously by gravity and vibration.


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>H</td>
<td>Cavity width</td>
</tr>
<tr>
<td>Q</td>
<td>Flux</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandt Number</td>
</tr>
<tr>
<td>R</td>
<td>Ratio Rav/Ra</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>Rav</td>
<td>vibrational Rayleigh number</td>
</tr>
<tr>
<td>Rρ</td>
<td>Density ratio</td>
</tr>
<tr>
<td>Rβ</td>
<td>Ratio of expansion coefficients</td>
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<tr>
<td>Rκ</td>
<td>Ratio of thermal conductivities</td>
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<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>u, v</td>
<td>Components of velocity fields</td>
</tr>
<tr>
<td>x, y</td>
<td>Dimensional coordinates</td>
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<td>Thermal diffusivity</td>
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<tr>
<td>μ</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>(pC)</td>
<td>Heat capacity</td>
</tr>
<tr>
<td>φ</td>
<td>Volume fraction</td>
</tr>
<tr>
<td>w</td>
<td>Pulse of vibrations</td>
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Indices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>f</td>
<td>Fluid</td>
</tr>
<tr>
<td>nf</td>
<td>Nanofluid</td>
</tr>
<tr>
<td>s</td>
<td>Solid</td>
</tr>
<tr>
<td>eff</td>
<td>effective</td>
</tr>
<tr>
<td>*</td>
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